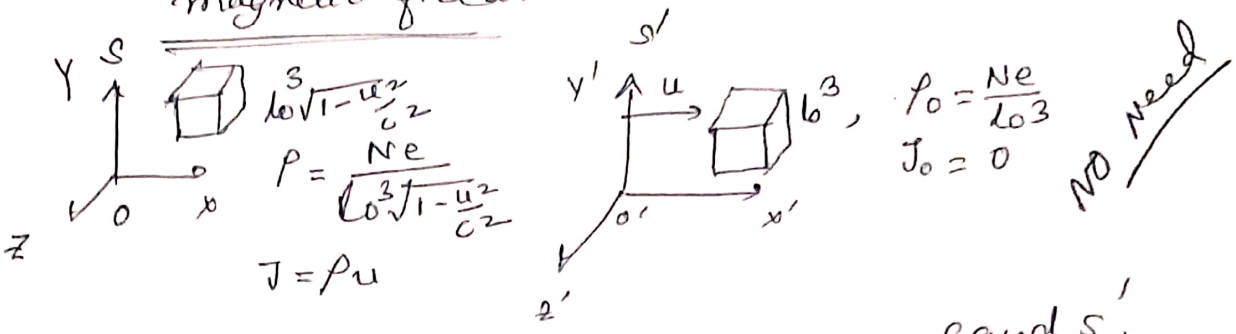
The interdependence of electric and magnetic field:



Let us consider two frames of references S and S'. S' is moving with const. translational velocity with respect to the frame S.

Consider a volume element containing some charge. Let us also consider that the volume element is a cube ~~cube~~ of side l_0 occupied by N number of charge, each of the charge e. ∓
So the volume is l_0^3 and it is at rest w.r. to S'.
In S', the charge density is

$$\rho_0 = \frac{Ne}{l_0^3} \longrightarrow (I)$$

Let the charges are also at rest w.r. to S'.
So there will be no current in S' and current density J_0, ~~vanish~~ with respect to S' is

$$J_0 = 0 \longrightarrow (II)$$

Now let's consider the volume element with respect to the S frame with respect to which the volume element moves with velocity u.

The direction of u is along the common edge of the cube.

$\therefore$ Volume of the cube w.r. to $S = l_0^3\sqrt{1-u^2/c^2}$ — (III)

$\therefore$ The charge density w.r. to S is $\rho = \frac{Ne}{l_0^3\sqrt{1-u^2/c^2}}$ — (IV)

And the current density $J = \rho u$ — (V)

Combining equation (I) and (IV) we have

$$\rho = \frac{\rho_0}{\sqrt{1-u^2/c^2}} \longrightarrow (VI)$$

As the charge are in motion with velocity u, w.r. to the frame S', so there is a current. The current density

$$J = \rho u = \frac{\rho_0 u}{\sqrt{1-u^2/c^2}} \longrightarrow (VII)$$

$$\left.\begin{aligned} J_x &= \frac{\rho_0 u_x}{\sqrt{1-u^2/c^2}} \\ J_y &= \frac{\rho_0 u_y}{\sqrt{1-u^2/c^2}} \\ \& \quad J_z &= \frac{\rho_0 u_z}{\sqrt{1-u^2/c^2}} \\ \rho &= \frac{\rho_0}{\sqrt{1-u^2/c^2}} \end{aligned}\right\} \quad (VIII)$$

From the equation (VII) the components of current density are considered as equation (VIII)

We have the relation between the momentum and energy is

$$E^2 = p^2c^2 + m_0^2c^4$$

$$\Rightarrow m^2c^2 - p^2 = m_0^2c^2$$

$$\Rightarrow \frac{E^2}{c^2} - (p_x^2 + p_y^2 + p_z^2) = m_0^2c^2 \rightarrow \text{an invariant}$$

$\longrightarrow$ (1 × a)

The relation between the space and time coordinate

$$c^2t^2 - (x^2 + y^2 + z^2) = c^2\tau^2 \rightarrow \text{an invariant}$$

$\longrightarrow$ (1 × b)

Considering the above analogies, the relation between the current density and charge density can be arranged in the following form of the equation.

$$c^2\rho^2 - (J_x^2 + J_y^2 + J_z^2) = c^2\rho_0^2$$

If m and m_0 be the masses of the charge particle

Then $\rho = \frac{\rho_0}{m_0} m$ & $J = \frac{\rho_0}{m_0} mu$

$= \frac{\rho_0}{m_0} p$ [$p = mu$]

& It is seen that J and ρ_0 transforms like p and m

∴ The set of transformation relation for J and ρ is

$$J_x' = \frac{J_x - v\rho}{\sqrt{1 - v^2/c^2}}$$

$$J_y' = J_y$$

$$J_z' = J_z$$

$$\rho' = \frac{\rho - vJ_x/c^2}{\sqrt{1 - \frac{v^2}{c^2}}}$$

(X)

[Considering the relative velocity between the frame is v]

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Q Mass & Energy:

K.E $= \gamma mc^2 - mc^2$

Relativistic momentum:

The relation between proper mass and rest mass is defined as and relativistic mass is equivalent.

$$m = \frac{m_0}{\sqrt{1 - \frac{v^2}{c^2}}}$$

$m_0 \rightarrow$ proper or rest mass

$m \rightarrow$ Relativistic mass.

∴ The relativistic momentum is defined as

$$P = \frac{m_0 v}{\sqrt{1 - v^2/c^2}} = mv \longrightarrow (1)$$

& Total energy

$$E = mc^2 = \frac{m_0 c^2}{\sqrt{1 - \frac{v^2}{c^2}}} \longrightarrow (11)$$

From equation (1) and (11) when $v \ll c$

and $m_0 = 0$

then $P = 0$, $E = 0$ } For a particle of zero rest mass and moving very low classical velocity.

But when $m = 0$, $V = C$

$E = \frac{0}{0}$ & $P = \frac{0}{0}$ } which is indeterminate, it may be anything. In this way relativity can explain the existance of high velocity massless particle.

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